6 DESIGN METHODS AND ANALYSIS

6.1 METHODS OF ANALYSIS

Second-order effects should be included in an analysis unless they can be proven to be insignificant. The P- Δ and the P- δ effects should be considered either in the analysis or in the design stage depending upon analysis method used.

The internal forces and moments acting on a structure may be calculated by one of the following analysis methods:

- (1) Simple design, lateral forces taken by linked rigid structure and beams are assumed simply supported on columns (see clause 6.5);
- (2) First-order linear elastic analysis, using the original and undeformed geometry of the structure (see clause 6.6);
- (3) Second-order elastic P-∆-only analysis, allowing for the effects of deformation of the structure (see clause 6.7);
- (4) Second-order elastic $P-\Delta-\delta$ analysis, allowing for the effects of deformation of the structure and the bowing deflection of members (see clause 6.8); and
- (5) Advanced analysis allowing for the effect of deformation of the structure and members and material yielding (see clause 6.9).



Figure 6.1 - Column effective length, P- Δ and P- δ moments

Both the P- Δ and P- δ effects with allowance for their initial imperfections must be allowed for either in the global analysis or in the design stage using clause 8.9.2.

The resistance of a structure is limited to the first plastic hinge for Class 1 plastic and Class 2 compact sections or to first yield for Class 3 semi-compact and Class 4 slender sections in methods (1) to (4), but to the elastic-plastic collapse load for method (5). Methods (3) and (4) are based on the large deflection analysis without and with allowance for member bow. Moment or force re-distribution due to material yielding is not allowed. Only Class 1 plastic and Class 2 compact sections can be used in method (5) with only Class 1 plastic section used for members possessing plastic hinges.

Static equilibrium, resistance to notional horizontal forces and sway stiffness should be checked using relevant and the most unfavourable and realistic load factors and combinations of load cases.

Superposition of moments and forces are allowed only in simple design and first-order linear elastic analysis methods.

Local member buckling effects due to flexural-torsional, torsional, and local plate buckling should be checked separately on a member basis, unless these effects have already been accounted for in the analysis or demonstrated to be negligible.

6.2 ANALYSIS MODELS AND ASSUMPTIONS

A suitable analysis model and consistent assumptions should be used to simulate the actual structural behaviour. The design of members and connections should accord with the analysis assumptions and not adversely affect the structural adequacy of other parts of the structure. All structures should be designed to have an adequate level of robustness against the effects of applied loadings and notional horizontal forces. Further, they should have sufficient sway stiffness and member stiffness to avoid the vertical loads producing excessive second-order P- Δ and P- δ effects. When this P- Δ effect is significant, it should be allowed for in column buckling strength, connection and beam design. The second-order P- δ effect should be considered for compression members.

The effective length of sloping members, inclined roofs and rafters under large axial forces may involve snap-through buckling and ambiguity in classifying the member as a beam or a column in effective length determination using charts. In such cases, the effective length cannot be determined by simple charts and second-order analysis or advanced analysis shall be used.

Ground-structure interaction can be considered by assuming the nominally rigid ground support to have stiffness equal to the column stiffness or to be pinned in other cases unless an assessment shows a more appropriate value of ground stiffness can be used.

6.3 FRAME CLASSIFICATION

6.3.1 General

Sway stiffness of a frame affects its buckling strength. The elastic critical load factor λ_{cr} determined in clause 6.3.2 can be used to classify a frame as sway, non-sway and sway ultra-sensitive frame for the following purposes.

- (1) Measure of sway stability for frame classification in this section and
- (2) Determination of moment amplification in clause 6.6.2

Frame classification can be carried out by calculating the elastic critical load factor λ_{cr} which can be obtained either by the eigenvalue analysis for general structures or the deflection method for geometrically regular and rectangular frame as described in clause 6.3.2.

6.3.2 Elastic critical load factor

6.3.2.1 General

Elastic critical load factor λ_{cr} can be obtained by the eigenvalue analysis or by the deflection method below. λ_{cr} of a frame is defined as the ratio by which the factored loads would have to be increased to cause elastic instability. Member imperfection is not required for frame classification.

6.3.2.2 Deflection method

For sway buckling mode of a geometrically regular and rectangular frame subjected to gravitational loads or gravitational loads plus horizontal load (e.g. wind), the elastic critical load factor λ_{cr} for a sway frame as shown in Figure 6.2 may be calculated as the smallest value for every storey with drift determined from the first-order linear analysis as,

$$\lambda_{cr} = \frac{F_N}{F_V} \frac{h}{\delta_N} \tag{6.1}$$

where F_V is the factored Dead plus Live loads on the floor considered;

- F_N is the notional horizontal force taken typically as 0.5% of F_V for building frames:
- *h* is the storey height; and
- δ_N is the notional horizontal deflection of the upper storey relative to the lower storey due to the notional horizontal force F_N .



Non-sway buckling mode

Sway buckling mode

Figure 6.2 Sway and Non-sway buckling modes

For non-rectilinear frame, the elastic critical load factor λ_{cr} can be obtained by the eigenvalue analysis. For single portal frame, the designer should refer to clause 8.11.

6.3.3 Non-sway frames

Except for advanced analysis, a frame is classified as non-sway and the $\text{P-}\Delta$ effect can be ignored when

(6.2)

For advanced analysis, a frame is classified as non-sway and the $\text{P-}\Delta$ effect can be ignored when

 $\lambda_{cr} \ge 15 \tag{6.3}$

6.3.4 Sway frames

Except for advanced analysis, a frame is classified as sway when

$10 > \lambda_{cr} \ge 5$	(6.4)
For advanced analysis, a frame is classified as sway when	
$15 > \lambda_{cr} \ge 5$	(6.5)

6.3.5 Sway ultra-sensitive frames

A frame is classified as sway ultra-sensitive when

 $\lambda_{cr} < 5 \tag{6.6}$

Only second order P- $\Delta\text{-}\delta$ or advanced analysis can be used for sway ultra-sensitive frames.

6.4 IMPERFECTIONS

6.4.1 General

In an analysis for members in compression or frames with members in compression, suitable allowance should be made for imperfections either in the analysis stage or in the design stage. Imperfections are due to geometrical and material effects and should be simulated by using suitable and equivalent geometrical imperfections.

Appropriate equivalent geometric imperfections may be used with suitable amplitudes and modes reflecting the combined effects of all types of imperfections.

The effects of imperfections shall be taken into account when considering the following:

- 1) Frame analysis
- 2) Member design
- 3) Bracing members

6.4.2 Frame imperfections

The effects of imperfections for typical structures shall be incorporated in frame analysis using an equivalent geometric imperfection in Equation 6.7 as an alternative to the notional horizontal force in clause 2.5.8,

 $\Delta=h\,/\,200$

(6.7)

where

h is the storey height;

 Δ is the initial deformation shown in Figure 6.1.

The shape of imperfection may be determined using the notional horizontal force in clause 2.5.8 or elastic critical mode in clause 6.4.4.

For regular multi-floor building frames, the shape may be simply taken as an inclined straight line.

These initial sway imperfections should be applied in all unfavourable horizontal directions, but need only be considered in one direction at a time.

For temporary works such as scaffolding, initial deformation should be taken as $\Delta = h / 100$. For demolition works, initial deformation equivalent to notional force specified in Code of Practice for Demolition of Buildings should be used.

The simulation of out-of-plumbness with notional horizontal force is indicated in Figure 6.3.



a) Imperfect frame geometry approach

b) Notional force approach

Figure 6.3 - Notional horizontal force for out-of-plumbness

6.4.3 Member imperfections

For a compression member, the equivalent initial bow imperfection specified in Table 6.1 may be used in second order analysis of the member.

Alternatively, the effects of imperfections can be considered in member design when using the effective length method and the moment amplification method in clause 8.9.2.

Buckling curves referenced in Table 8.7	$rac{m{e}_0}{L}$ to be used in Second-order P- Δ - δ elastic analysis
a ₀	1/550
а	1/500
b	1/400
С	1/300
d	1/200

Table 6.1 - Values of member initial bow imperfection used in design

In Table 6.1, e_0 is the amplitude of the initial bow imperfection. Variation of the initial bow imperfection v_0 along the member length is given by,

$$v_0 = e_0 \sin \frac{\pi x}{l} \tag{6.8}$$

L is the member length,

x is the distance along the member.

6.4.4 Elastic critical mode

For non-rectilinear frames whereby the method of notional horizontal force in clause 2.5.8 is inapplicable, the elastic buckling mode can be used to simulate the global imperfections. The amplitude of such global imperfection can be taken as building height / 200 for permanent structures or height / 100 for temporary structures. When the second order analysis is determined for use in structural analysis, the structure should be checked for stability and buckling strength using first global elastic buckling mode. Local elastic buckling mode of secondary member should not be used in place of the global elastic buckling mode.

6.5 SIMPLE DESIGN

Sway is prevented by connections and ties to a relatively rigid frame. Simple design can only be used when λ_{cr} of the frame system is not less than 10. The connections between members and columns are assumed to be pinned and the moment developed will not adversely affect the structural adequacy and robustness of the members or the structure. Sufficient connection flexibility should be designed and detailed. Sway stability should be provided by structures with adequate stiffness and strength to resist the lateral wind or notional horizontal force. Realistic assumption for eccentricity of reaction should be made in design. A minimum eccentricity of 100 mm between beam reaction and the face of column or the centre of stiff bearing length should be assumed.

6.6 FIRST-ORDER LINEAR ELASTIC ANALYSIS (FIRST-ORDER INDIRECT ANALYSIS)

6.6.1 General

P-∆ and P-δ effects should be checked in the member design by the moment amplification and the effective length methods. The first order linear elastic analysis method can only be used in rectilinear non-sway frames whereby the elastic critical load factor $\lambda_{cr} \ge 10$ and for rectilinear sway frames with $\lambda_{cr} \ge 5$. For the calculation of elastic critical load factor λ_{cr} , Equation 6.1 should only be used for rectilinear frames subjected to gravitational loads or gravitational loads plus horizontal loads (e.g. wind). For design of beam-columns, clause 8.9.2 should be referred to.

The indirect analysis referred in clauses 6.6 and 6.7 carries out an analysis without full consideration of geometric imperfections and a separate individual member design is necessary for a safe design.

6.6.2 Moment amplification for sway frames

The bending moment due to horizontal load is amplified by the following factor.

$$\frac{\lambda_{cr}}{\lambda_{cr}-1}$$

(6.9)

where

 λ_{cr} is the elastic critical load factor determined in clause 6.3.2.

Connections and connecting members should be designed using the amplified bending moment allowing for first-order and second-order $P-\Delta$ moments.

The P- Δ effect and moment amplification can be ignored for member and connection design when λ_{cr} is not less than 10.

6.6.3 Effective length for sway and non-sway frames

The effective length of a sub-frame in a continuous frame shown in Figure 6.4 can be determined using the distribution factors, k_1 and k_2 , at the column ends as,





where

 K_1 and K_2 are the values of K_c for the adjacent column lengths; K_{11} , K_{12} , K_{21} and K_{22} are the values of K_b for the adjacent beams; K_b , stiffness beam, should be taken as zero for the beam not to exist.

Figure 6.4 - Restraint coefficients for sub-frame

The effective length factor can be determined from Figures 6.5a and 6.5b using k_1 and k_2 determined from equation 6.10.



Figure 6.5a - Effective length factor (L_E/L) for sway frames



Figure 6.5b - Effective length factor (L_E/L) for non-sway frames

The stiffness of the beams should be taken from Table 6.2 with consideration of the following effects.

- When the moment at one end of the column exceeds 90% of its reduced moment capacity under the axial force, the value of *k* is taken as unity.
- When the axial force in restraining beams is significant, the stiffness reduction should be considered.
- When beams are linked to the joint by semi-rigid connections, the beam stiffness should be revised as,

$$k'_{b} = \frac{k_{b}k_{sp}}{k_{b} + k_{sp}} \tag{6.11}$$

where

 k'_{b} and k_{b} are respectively the revised and the original beam stiffnesses and k_{sp} is the connection stiffness obtained from a laboratory test or the literature.

Table 6.2 - Stiffness of beam for effective length determination in continuous structures

Loading conditions of the beam	Non-sway mode	Sway mode
Beams supporting concrete or composite floor	1.0 /	$1.0\frac{l}{L}$
Other beams under load along span	$0.75 \frac{l}{L}$	$1.0\frac{l}{L}$
Other beams under end moments only	$0.5\frac{l}{L}$	$1.5\frac{l}{L}$

Note: *I* is the second-moment of area about the buckling axis and

L is the member length.

6.6.4 Maximum slenderness ratio

The slenderness ratio should be limited to 200 for members in compression and 300 for members in tension except when a second-order analysis allowing for self weight and other member loads of the member is used or measures are taken against detrimental effects due to high slenderness.

6.7 SECOND-ORDER P-∆-ONLY ELASTIC ANALYSIS (SECOND-ORDER INDIRECT ANALYSIS)

6.7.1 General

This analysis method considers the changes in nodal coordinate and sway such that the P- Δ effect is accounted for. The effect of member bowing (P- δ) is not considered here and should be allowed for separately. Member resistance check for P- δ effect to clause 8.7 is required and this P- Δ -only method of analysis and design is under the same limitations of use as the linear analysis.

6.7.2 Method of analysis

The analysis can be carried out by allowing change of nodal coordinates of a structure to assess the non-linear effects. The member effective length factor should be taken as 1 and the moments induced at member ends and along members should be amplified according to clause 8.9.2.

6.7.3 Applications and limitations

The second order P- Δ -only elastic analysis calculates the sway-induced P- Δ moment as an alternative to the amplification moment computed in clause 6.6.2. The connections and connecting members should be designed using this refined bending moment, which allows for first-order bending moment and the second-order P- Δ moments. The column buckling resistance should be determined to clause 8.9.2.

6.8 SECOND-ORDER $P-\Delta-\delta$ ELASTIC ANALYSIS (SECOND-ORDER DIRECT ANALYSIS)

6.8.1 General

Both the P- Δ and P- δ effects are accounted for in the computation of bending moment in this method. Checking the buckling resistance of a structure to clause 6.8.3 is sufficient and member check to clause 8.9.2 is not needed. The direct analysis here allows an accurate determination of structural response under loads via the inclusion of the effects of geometric imperfections and stiffness changes directly in the structural analysis and equations 6.12 to 6.14 for section capacity check in the structural analysis are sufficient for structural resistance design.

6.8.2 Method of analysis

A second-order P- Δ - δ elastic analysis considers the followings.

- (1) Equilibrium in the deformed position of the structure (i.e. $P-\Delta$ effect);
- (2) Member bowing deflection and stiffness change (i.e. $P-\delta$ effect); and
- (3) Frame and member imperfections in clause 6.4.

6.8.3 Applications and limitations

The method is limited to the capacity at the load level where the first plastic hinge for Class 1 and Class 2 is formed or the maximum stress at extreme fibre for Class 3 and Class 4 reaches the design strength. Except for Class 4 slender sections, the sectional capacity should be checked using the following equation allowing for the P- Δ and P- δ effects.

$$\frac{F_c}{A_g \rho_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{F_c}{A_g \rho_y} + \frac{\overline{M}_x + F_c(\Delta_x + \delta_x)}{M_{cx}} + \frac{\overline{M}_y + F_c(\Delta_y + \delta_y)}{M_{cy}} \le 1$$
(6.12)

in which

 δ_x, δ_y are the member deflections due to member initial bow and loads on the member and about x- and y-axes.

As an alternative to Equation 6.12, section capacity could be checked by the following reduced moment capacities equation.

$$\left(\frac{M_x}{M_{rx}}\right)^{z_1} + \left(\frac{M_y}{M_{ry}}\right)^{z_2} = \left(\frac{\overline{M}_x + F_c(\Delta_x + \delta_x)}{M_{rx}}\right)^{z_1} + \left(\frac{\overline{M}_y + F_c(\Delta_y + \delta_y)}{M_{ry}}\right)^{z_2} \le 1$$
(6.13)

in which M_{rx} and M_{ry} are reduced moment capacities about x- and y-axes by assuming the area nearest to centroid of the cross section would take the axial load F_c with the remaining area used in computing moment resistances M_{rx} and M_{ry} .

 z_1 and z_2 are constants taken as follows.

For I- and H- sections with equal flanges,

$$z_1 = 2.0$$
 $z_2 = 1.0$

For solid and hollow circular sections,

 $z_1 = z_2 = 2.0$

For solid and hollow rectangular sections,

$$z_1 = z_2 = 1.6$$

For all other sections,

 $z_1 = z_2 = 1.0$

For Class 4 slender cross-sections, the effective area A_{eff} should be used in place of the gross sectional area in Equations 6.12 and 6.13.

Member lateral-torsional and torsional buckling checks are carried out separately or alternatively by replacing M_{cx} in the above equation by the buckling resistance moment M_b in Equations 8.20 to 8.22. If moment equivalent factor m_{LT} is less than 1, both Equation 6.12 or 6.13 and Equation 6.14 are required for member resistance check.

$$\frac{F_c}{A_g p_y} + \frac{M_x}{M_{cx}} + \frac{M_y}{M_{cy}} = \frac{F_c}{A_g p_y} + \frac{m_{LT} [\overline{M}_x + F_c (\Delta_x + \delta_x)]}{M_b} + \frac{m_y [\overline{M}_y + F_c (\Delta_y + \delta_y)]}{M_{cy}} \le 1$$
(6.14)

The equivalent uniform moment factor m_{LT} for beams and the moment equivalent factor m_y for flexural buckling can be referred to Tables 8.4 a & b and Table 8.9.

For members in bending and sensitive to buckling, imperfection on both axes should be considered if effective length has reduction in capacity about buckling in both axes.

6.9 ADVANCED ANALYSIS

6.9.1 General

Advanced analysis may be used when the design load induces plasticity in a structure. Instability, $P-\Delta$ and $P-\delta$ effects, and frame and member initial imperfections should be accounted for so that the non-linear structural behaviour can be captured in the analysis.

In advanced analysis, the strength, the stiffness and the ductility limits must be satisfied. The member cross-sections shall be Class 1 plastic section for members containing plastic hinges or at least Class 2 compact section for members without being designed for formation of plastic hinges. Frames made of other sections may also be designed by the advanced analysis when the local buckling effects are properly accounted for.

When advanced plastic analysis is used, the frame should be effectively restrained against significant displacement out of the plane of the frame or out-of-plane buckling is checked for individual members. Unless the effect of out-of-plane buckling has been considered, lateral restraint shall be provided at all plastic hinge locations. The restraint should be provided within a distance along the member from the theoretical plastic hinge location not exceeding half the depth of the member. The effects due to residual stress, erection procedure, interaction with foundation and temperature change should be taken into account in the analysis.

6.9.2 Method of analysis

A full second-order plastic hinge or plastic zone analysis can be used in an Advanced Analysis. The method should consider the followings.

- (1) Equilibrium in the deformed position of the structure;
- (2) Member bowing deflection and stiffness change;
- (3) Frame and member imperfections in clause 6.4;
- (4) Material yielding by plastic hinge or plastic zone models; and
- (5) Ductility adequacy in plastic hinges (i.e. The rotation capacity should be larger than the rotation at maximum design moment or moment in the design range).

6.9.3 Applications and limitations

The method only directly locates the maximum limit load of a structure allowing for the formation of plastic hinges and under different load cases.

6.10 BRACING MEMBERS

The shape of imperfection should be determined using the elastic critical mode with the amplitude of initial bowing imperfection as:

$$e_o = k_r L / 500$$

where L is the span of the bracing system and

(6.15)

$$k_r = \sqrt{0.2 + \frac{1}{n_r}}$$
 but $k_r \le 1.0$

(6.16)

where n_r is the number of members being restrained by the bracing member.

6.11 CONNECTION CLASSIFICATION IN ANALYSIS

Connections can be classified as pinned, rigid or semi-rigid, depending on their strength, stiffness and rotational capacity. The detailing and design of connections must be consistent with the assumptions used in the calculation.

The strength and stiffness capacity of connections should be consistent with the assumptions made in the analysis and design. For advanced analysis, rotational capacity should also satisfy the rotational requirement in analysis.

6.11.1 Pinned connections

A pinned connection shall be designed and detailed to avoid development of significant moment affecting adversely the members of the structure. It should also be capable of transmitting forces calculated in the design and allowing for sufficient rotations.

6.11.2 Rigid connections

A rigid connection shall be designed and detailed such that its deformation will not adversely affect the force and moment distribution and stiffness of members in the frame. It should be able to transmit forces and moments calculated in design.

6.11.3 Semi-rigid connections

Semi-rigid connections include those connections which cannot satisfy the pinned or rigid connections requirements in clauses 6.11.1 and 6.11.2. The moment and deformation of a semi-rigid connection should not adversely affect members in a frame and the connection should be modeled in an analysis indicated in Figure 6.6 with connection stiffness obtained from literatures or by tests. It should be designed and detailed to have the capacity to transmit calculated forces and moment and to deform suitably to accept the rotation.



Rotation, ϕ

Figure 6.6 - Connection behaviour